

# Coriolis force in geophysics: an elementary introduction and examples

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**Abstract.** We show how geophysics can illustrate and thus improve classical mechanics lectures concerning the study of Coriolis force effects. Here, we are interested in atmospheric and oceanic phenomena, with which we are familiar, and which are for that reason of pedagogical and practical interest. Our aim is to model them in a very simple way to bring out the physical phenomena that are involved.

## 1. Introduction

The study of mechanics in non-inertial frames, for which the Coriolis force effects for the rotating Earth form the paradigm, are often restricted to the standard examples of a deflected projectile and the Foucault pendulum. In these two examples, the Coriolis force is only a *small perturbation*. In order to make the Coriolis force effects dominant, one must consider phenomena taking place at the geophysical scale.

This is the approach followed by the present paper. The first section is devoted to the presentation of the basic equations. In the second section, we discuss the physics of anticyclones and cyclones: we describe their rotation in the geostrophic approximation, and show how cyclones (but not anticyclones) may become hurricanes. The third section presents a second example of atmospheric circulation, the jet streams, which are stabilized by the Coriolis force. We also show that these strong winds are associated with Rossby waves. The last section presents two examples of oceanic circulation: wind-driven circulation and Kelvin waves.

## 2. Basic equations

### 2.1. Navier–Stokes equation in a rotating frame

Let us introduce two frames  $\mathcal{R}$  and  $\mathcal{R}'$  in relative motion. The inertial frame  $\mathcal{R}$  is the geocentric one, with origin at the centre of the Earth,  $O$ , and whose axes are along directions of fixed stars. The frame  $\mathcal{R}'$  is fastened to the Earth. It has an angular velocity  $\Omega$  with respect to  $\mathcal{R}$ , where  $\Omega$  is the angular velocity of rotation of the Earth ( $\Omega \simeq 7.29 \times 10^{-5} \text{ rad s}^{-1}$ ). The following relation between the acceleration of a point  $M$ ,  $a_{\mathcal{R}}(M)$  in  $\mathcal{R}$ , and  $a_{\mathcal{R}'}(M)$  in  $\mathcal{R}'$  may easily be obtained [1]:

$$a_{\mathcal{R}}(M) = a_{\mathcal{R}'}(M) + 2\Omega \wedge v_{\mathcal{R}'}(M) + \Omega \wedge (\Omega \wedge OM). \quad (2.1)$$

In equation (2.1), the term  $2\boldsymbol{\Omega} \wedge \mathbf{v}_{\mathcal{R}'}(M)$  is the Coriolis acceleration,  $\mathbf{v}_{\mathcal{R}'}(M)$  is the velocity of  $M$  in  $\mathcal{R}'$ , and  $\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{OM})$  is the centrifugal acceleration. In the rotating frame  $\mathcal{R}'$ , the Navier–Stokes equation takes into account the above inertial terms and reads [2]

$$\frac{\partial \mathbf{v}_{\mathcal{R}'}}{\partial t} + (\mathbf{v}_{\mathcal{R}'} \cdot \nabla) \mathbf{v}_{\mathcal{R}'} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \mathbf{f} - 2\boldsymbol{\Omega} \wedge \mathbf{v}_{\mathcal{R}'} - \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{OM}) + \nu \Delta \mathbf{v}_{\mathcal{R}'}. \quad (2.2)$$

The force  $\mathbf{f}$  includes the gravitational force and other external forces if they exist,  $\rho$  is the density of the fluid and  $p$  the pressure field. The dependence on  $M$  has been removed in all the terms for clarity. The centrifugal force is conservative. If this is also the case for  $\mathbf{f}$ , one can rewrite the terms  $\nabla p$ ,  $-\mathbf{f}$  and  $\rho \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{OM})$  as  $\nabla p'$ , where  $p'$  is called the *dynamical pressure*. In the rotating frame, the hydrostatic equilibrium equation is  $\nabla p' = \mathbf{0}$ . The dynamical pressure  $p'$  reads, within a constant,  $p' = p + \rho g z$ , where  $g$  is the Earth gravity field. Recall that  $g$  includes the centrifugal term, and is thus slightly different from the gravitational field, which only takes into account the Earth's attraction [1].

## 2.2. Reynolds and Rossby numbers

The nonlinearity of the Navier–Stokes equation makes it difficult to solve in general. It is hence necessary to evaluate the relative importance of the different terms in order to make further simplifications. This is done by introducing the different characteristic scales of the flow:  $L$  denotes the typical length scale,  $U$  the velocity,  $\boldsymbol{\Omega}$  the angular velocity and  $\nu$  the kinematic viscosity. Two dimensionless numbers may then be derived from these scales.

(a) The Reynolds number is defined as

$$Re = \left| \frac{(\mathbf{v} \cdot \nabla) \mathbf{v}}{\nu \Delta \mathbf{v}} \right| = \frac{U^2/L}{\nu U/L^2} = \frac{UL}{\nu}. \quad (2.3)$$

It characterizes the relative importance of the momentum transport in the fluid by advection and viscous diffusion. For the atmospheric flows studied here, typical values are:  $U \sim 10 \text{ m s}^{-1}$ ,  $L \sim 10 \text{ km}$  and  $\nu \sim 10^{-5} \text{ m}^2 \text{ s}^{-1}$ . Thus, the Reynolds number is  $\approx 10^{10}$ . A large value of the Reynolds number is also obtained for oceanic flows<sup>†</sup>. Hence, the Navier–Stokes equation reduces, for geophysical flows, to the Euler equation:

$$\frac{\partial \mathbf{v}_{\mathcal{R}'}}{\partial t} + (\mathbf{v}_{\mathcal{R}'} \cdot \nabla) \mathbf{v}_{\mathcal{R}'} = -\frac{1}{\rho} \nabla p + \frac{1}{\rho} \mathbf{f} - 2\boldsymbol{\Omega} \wedge \mathbf{v}_{\mathcal{R}'} - \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{OM}). \quad (2.4)$$

Moreover, geophysical flows are turbulent (high Reynolds number) [3]. For simplicity, we ignore this complication in what follows. A simple way of taking into account the relevant effects of turbulence will be presented in the last section (see also section 5.1).

(b) The Rossby number is defined as

$$Ro = \left| \frac{(\mathbf{v} \cdot \nabla) \mathbf{v}}{2\boldsymbol{\Omega} \wedge \mathbf{v}} \right| = \frac{U^2/L}{\Omega U} = \frac{U}{L\Omega}. \quad (2.5)$$

It compares the advection and the rotation effects. The Coriolis force dominates if  $Ro \ll 1$ . A geophysical flow, characterized by a large length scale, may easily be influenced by the Earth's rotation, since one typically has  $Ro \sim 10^{-2} \ll 1$ . On the other hand, an emptying bath for which  $U \sim 1 \text{ m s}^{-1}$ , and  $L \sim 10^{-1} \text{ m}$ , has  $Ro \sim 10^5$ . Such a flow is more strongly influenced by the advection in the fluid, and thus by the initial conditions, than by the Earth's rotation.

<sup>†</sup> For oceans,  $Re \sim 10^{11}$ , with  $U \sim 1 \text{ m s}^{-1}$ ,  $\nu \sim 10^{-6} \text{ m}^2 \text{ s}^{-1}$ : the approximation is thus the same.

### 3. Atmospheric eddies

#### 3.1. Anticyclones and cyclones

We first consider the situation when the Rossby number is negligible. This is the case for anticyclones and cyclones since one typically has  $U \sim 10 \text{ m s}^{-1}$ ,  $L \sim 1000 \text{ km}$ , which yields  $Ro \sim 0.1$ . In the Euler equation (2.4), we only have to keep the gravity, pressure and Coriolis terms. This hypothesis constitutes the *geostrophic approximation*. For each point  $M$  of the Earth, we define a vertical axis ( $Mz$ ), and a cylindrical coordinate system  $(r, \theta, z)$ . The vertical component of the velocity field is assumed to be zero, which implies that the movements of the fluid are locally horizontal.  $u$  is the radial component of the velocity field and  $v$  the tangential one. The Earth's angular velocity  $\Omega$  is written as  $\Omega = \Omega_{\parallel} + \Omega_{\perp}$  where  $\Omega_{\parallel} \equiv \Omega \sin \lambda \mathbf{u}_z$  and  $\Omega_{\perp}$  is the projection  $\Omega$  on the plane  $(r, \theta)$ ;  $\lambda$  is the latitude. The flow is assumed to be stationary. In this system of coordinates, the Euler equation can be rewritten, under the geostrophic approximation, as

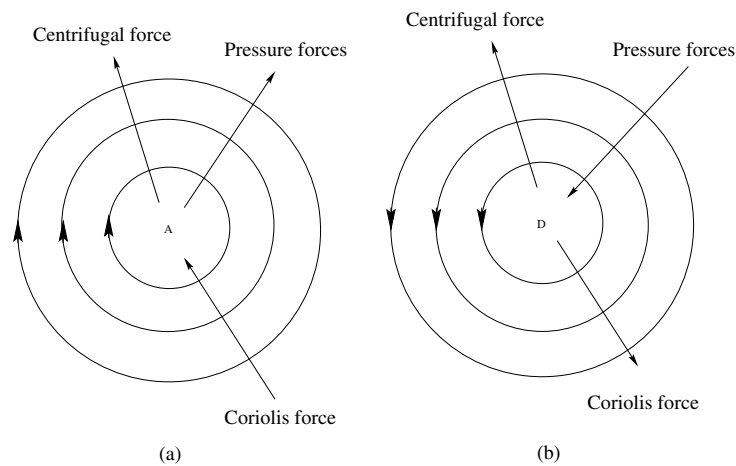
$$\frac{\partial p}{\partial r} = \rho v f \quad (3.1a)$$

$$\frac{1}{r} \frac{\partial p}{\partial \theta} = -\rho u f \quad (3.1b)$$

$$\frac{\partial p}{\partial z} = -\rho g - 2\rho(\Omega_{\perp} \wedge v) \cdot \mathbf{u}_z. \quad (3.1c)$$

In these equations,  $f \equiv 2\Omega \sin \lambda$  is the *Coriolis parameter*. In equation (3.1c), the term  $2\rho(\Omega_{\perp} \wedge v) \cdot \mathbf{u}_z$  is small compared with  $\rho g$  ( $\Omega U/g \sim 10^{-5}$ ). Equation (3.1c) therefore reduces to the hydrostatic equilibrium equation  $\partial p/\partial z = -\rho g$ .

If we consider the case of an eddy in the Northern Hemisphere and assume that the velocity field is tangential ( $u = 0$ ), then,  $v < 0$  (clockwise rotation) implies  $\partial p/\partial r < 0$ . The pressure is higher at the eddy centre than outside: it is an anticyclone. A cyclone would correspond to an anticlockwise rotation. Both situations are represented in figure 1. The directions of rotation are opposite in the Southern Hemisphere.



**Figure 1.** Anticyclone (a) and cyclone (b) in the Northern Hemisphere.

We end this section with two concluding remarks about the presence of the Coriolis force.

- (a) Without this force, an eddy centre is *always* a pressure minimum. However, in the case of the anticyclone, the Coriolis force stabilizes the inverse situation, with the eddy centre being a pressure maximum.

- (b) In its vectorial form, the geostrophic equilibrium equation reads:  $\nabla p' = -2\rho\Omega \wedge v$ . This implies that the pressure  $p'$  is constant along a streamline. When the usual Bernoulli equation is valid, pressure variations are, conversely, associated with velocity variations along a streamline.

### 3.2. Hurricanes

Let us consider an eddy (anticyclone or cyclone) whose angular velocity and radius are  $\omega$  and  $R$ , respectively. The Rossby number characterizing this eddy can be written as  $Ro = U/L\Omega = \omega/\Omega$ . Therefore, the geostrophic equilibrium corresponds to a small angular velocity of the eddy, i.e.  $\omega \ll \Omega$ . We shall now consider the case where the eddy's angular velocity is not small compared with the Earth's rotation. This means that the centrifugal force due to the eddy rotation has to be taken into account. In this case, the Rossby number is of order unity. In the frame  $\mathcal{R}'$ , the fluid particle has a uniform circular motion. Forces acting on it are the Coriolis force and the radial pressure gradient. The equation of motion for a fluid particle, located at the eddy's periphery reads, in  $\mathcal{R}'$ :

$$-r_0\omega^2 = -\frac{1}{\rho} \frac{dp}{dr} + r_0 f \omega \quad (3.2)$$

where  $r_0$  is the eddy radius. The term  $-r_0\omega^2$  corresponds to the centrifugal acceleration of the fluid particle, and  $r_0 f \omega$  is the Coriolis term.

An anticyclone in the Northern Hemisphere is shown in figure 1(a). For such an equilibrium, the Coriolis force compensates both pressure and centrifugal forces. If the angular velocity of the anticyclone grows, the Coriolis force is not sufficient to counterbalance these two forces since the centrifugal force grows faster than the Coriolis force with increasing  $\omega$ . This is not the case for the cyclone depicted in the figure 1(b). The pressure and centrifugal forces may counterbalance each other when the rotation of the cyclone becomes faster. This qualitative approach shows that there is no limit to the kinetic energy of rotation for a cyclone.

More quantitatively, equation (3.2) can be solved to find

$$\omega_{\pm} = \frac{f}{2} \left( -1 \pm \sqrt{1 + \frac{G}{G_0}} \right) \quad (3.3)$$

where  $G \equiv dp/dr$  and  $G_0 \equiv \rho r_0 f^2/4$ . Figure 2 gives the evolution of an eddy angular velocity as a function of the radial pressure gradient. In this figure, the geostrophic situation can be found around the origin (small pressure gradient and angular velocity). In the Northern Hemisphere, the sign of the angular velocity is the same as that of the pressure gradient. One can even obtain the angular velocity of an eddy by expanding the expression (3.3) around zero:  $\omega \approx fG/4G_0$ .

The condition  $G > -G_0$ , for the existence of the above solutions, gives a limit to the angular velocity of an anticyclone ( $G < 0$ ). One finds  $\omega_{\max} = 2\Omega \sin \lambda$ . Such a limit does not exist for a cyclone ( $G > 0$ ). When the angular velocity grows, the radial pressure gradient follows this evolution and becomes more and more important. This explains why hurricanes are always associated with very low pressure.

We note, in conclusion, that the balance between the centrifugal force and the radial pressure gradient is possible whatever the direction of rotation. Thus, the existence of clockwise hurricanes in the Northern Hemisphere cannot be excluded. However, most of the hurricanes observed in the Northern Hemisphere are anticlockwise and result from the amplification of earlier tropical cyclones, the amplification mechanism being the conversion of the latent heat of evaporating tropical warm waters into rotational kinetic energy.

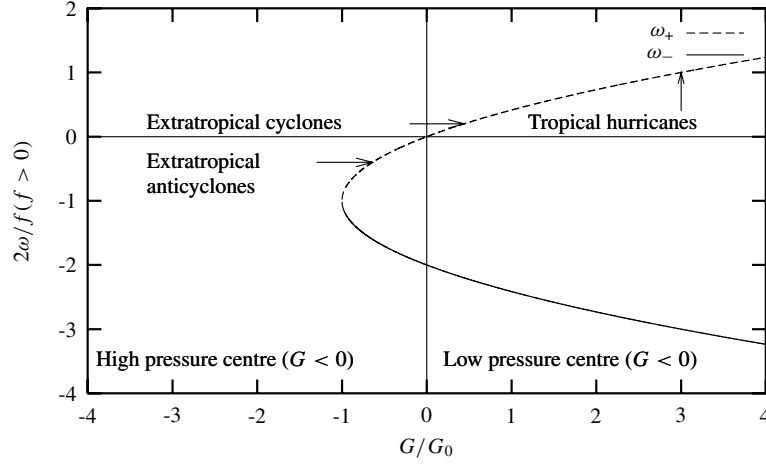


Figure 2. Normalized angular velocity as a function of the normalized pressure gradient.

#### 4. Jet streams and Rossby waves

The difference in solar heating between the equatorial and polar regions drives a convective cell at the planetary length scale, the *Hadley cell*, which extends in both hemispheres from the Equator up to the subtropical latitudes. The heated equatorial air rises, moves towards the Poles where it cools, then sinks and returns to the Equator. When returning, the air is deflected towards the West by the Coriolis force, generating easterly winds at the tropical latitudes which are known as the *Trade Winds*. Conversely, the upper-troposphere trajectories towards the Poles are deflected towards the East. Because of the thermal structure of the atmosphere [4], these upper-level westerly winds concentrate in narrow tubes of very strong winds of up to  $80 \text{ m s}^{-1}$ , the *jet streams*. The jet streams are typically found at altitudes of  $\approx 10 \text{ km}$  and at latitudes of between  $30^\circ$  and  $40^\circ$ . However, their strength and location may depart significantly from these mean values, depending on the season, the longitude, and the day-to-day thermal structure of the atmosphere at middle latitudes. It can be noted that Piccard and Jones took great advantage of the jet streams for their recent travel around the world in a balloon. The jet streams are also useful to aircraft flying from North America to Europe.

In this section, we propose to show how a *zonal* wind (i.e. along the parallels) may be stabilized by the Coriolis force. A mass of air,  $M$ , near the Earth's surface is reduced to a point  $G$ . Its coordinates are the usual spherical ones  $(R, \theta, \varphi)$ ,  $\theta$  being the colatitude and  $R$  the radius of the Earth. The velocity of  $G$  can then be explicitly written as  $\mathbf{v}_{\mathcal{R}'}(G) = R\dot{\theta}\mathbf{u}_\theta + R\dot{\varphi}\sin\theta\mathbf{u}_\varphi$ . The quantity  $R\dot{\varphi}\sin\theta$  is the drift velocity  $u_0$  of the point  $G$  along a parallel. We deduce the following expression for the Coriolis force moment about the centre of the Earth (point  $O$ ):

$$\mathcal{M}_O = 2MR^2\Omega\dot{\theta}\mathbf{u}_\theta + 2MR\Omega u_0 \cos\theta\mathbf{u}_\varphi. \quad (4.1)$$

The computation of the angular momentum of  $G$  about  $O$ , in the frame  $\mathcal{R}'$ , yields

$$\mathbf{L}_{\mathcal{R}'}(O) = -(MR^2)\dot{\varphi}\sin\theta\mathbf{u}_\theta + (MR^2)\dot{\theta}\mathbf{u}_\varphi. \quad (4.2)$$

The theorem of angular momentum for the point  $G$ , about  $O$  and projected on  $\mathbf{u}_\varphi$  gives

$$-\ddot{\lambda} = 2\frac{\Omega u_0}{R}\sin\lambda \quad (4.3)$$

where  $\lambda \equiv \pi/2 - \theta$  is the latitude. This equation is linearized for small deviations around a given latitude  $\lambda_0$ , leading to

$$\delta\ddot{\lambda} + \left[ 2\frac{\Omega u_0}{R}\cos\lambda_0 \right] \delta\lambda = 0 \quad (4.4)$$

where  $\delta\lambda \equiv \lambda - \lambda_0$ . The meridional motion of  $G$  remains bounded, only if  $u_0 > 0$ , which corresponds to a drift velocity from West to East. This motion is characterized by small oscillations around the mean latitude  $\lambda_0$  with angular frequency  $\omega_0 = \sqrt{2\Omega u_0 \cos \lambda_0}/R$ . These oscillations correspond to the *stationary case of a Rossby wave* [5]. More generally, Rossby waves in the atmosphere are guided by strong westerly winds.

## 5. Oceanic circulation

Oceanic circulation is, of course, described by the same equations as atmospheric circulation. For large-scale oceanic currents, such as, for example, the Gulf Stream, the geostrophic approximation (see section 3.1) is relevant: the Coriolis force compensates the horizontal pressure gradient, which is related to the slope of the free surface, which is not necessarily horizontal [6].

We shall be interested here in a slightly different case for which the interaction between the wind and the ocean gives rise to a current.

### 5.1. Wind-driven circulation: Ekman transport

The wind induces a friction at the ocean surface, transmitted through turbulence to the deeper layers of the sea. There is a supplementary difficulty that we cannot ignore here. The flow is not laminar, but essentially turbulent. The fluid viscosity is related to molecular agitation, dissipating the energy of a fluid particle. A diffusive momentum transport is associated with this phenomenon. In a turbulent flow, agitation dissipates the energy associated with the mean velocity of the current. This analogy allowed Prandtl to introduce the notion of an *eddy viscosity* [3]. In this approximation, considering  $v_{\mathcal{R}}$  as the mean flow velocity, the Navier–Stokes equation (2.2) remains unchanged, the eddy viscosity  $\nu_{\text{turb}}$  being added to the kinematic viscosity  $\nu$ . It must be noted that the former is a property of the flow, while the latter is a property of the fluid. As far as geophysical flows are concerned, the kinematic viscosity is neglected since, typically,  $\nu_{\text{turb}}/\nu \sim 10^5$  for oceanic flows, and  $\nu_{\text{turb}}/\nu \sim 10^7$  for atmospheric flows.

Let us write the Navier–Stokes equation in projection on  $(Oxyz)$ , where  $(Oxy)$  is the surface of the globe,  $(Oz)$  the ascendant vertical, and  $(u, v, w)$  are the velocity components:

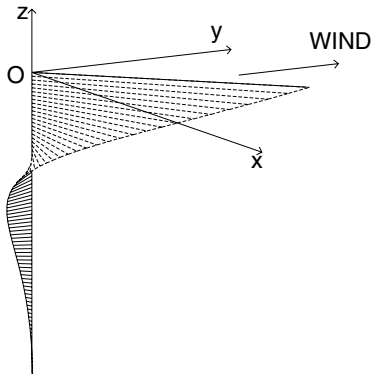
$$\begin{aligned} \frac{du}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + fv + \nu_{\text{turb}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{dv}{dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} - fu + \nu_{\text{turb}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right). \end{aligned} \quad (5.1)$$

For a stationary situation with large Rossby number, the acceleration terms are negligible: the velocity then depends on space only. The horizontal-pressure-gradient terms can also be neglected since the equations have been linearized and one can consider the real physical situation as the superposition of a geostrophic current (taking into account the pressure terms) and a wind-driven current, which will now be described. We consider a solution depending on space only through the coordinate  $z$ . The boundary conditions are the following: the velocity must be finite, both as  $z \rightarrow -\infty$  and at the free surface, the stress is proportional to  $\partial v/\partial z$  and parallel to the wind flow, assumed to be in the  $(Oy)$  direction. One can solve equation (5.1) and find the velocity field (the solution is straightforward, by defining  $W(z) \equiv u(z) + iv(z)$ ):

$$u(z) = \pm V_0 \cos\left(\frac{\pi}{4} + \frac{z}{\delta}\right) \exp\left(\frac{z}{\delta}\right) \quad v(z) = V_0 \sin\left(\frac{\pi}{4} + \frac{z}{\delta}\right) \exp\left(\frac{z}{\delta}\right) \quad (5.2)$$

where  $\delta \equiv \sqrt{2\nu_{\text{turb}}/|f|}$  is a distance called the *Ekman depth*. Typical values for  $\delta$  are  $\sim 10$ – $100$  m. The ‘+’ sign relates to the Northern Hemisphere and ‘–’ to the Southern Hemisphere.

Close to the surface ( $z = 0$ ), the current deviates by  $45^\circ$ , and the direction of the velocity rotates clockwise (anticlockwise) in the Northern (Southern) Hemisphere. The amplitude of



**Figure 3.** Ekman spiral. The surface is generated by the velocity field  $v(0, 0, z)$ .

the velocity decreases exponentially on a length scale  $\delta$ , which represents the characteristic depth over which the influence of the wind is significant. This velocity field, the so-called *Ekman spiral*, is plotted in figure 3. The mean effect of the wind, over a depth  $\delta$ , is the fluid motion in a direction perpendicular to it: this effect is called the *Ekman transport*.

### 5.2. Kelvin waves

The main difference between atmospheric flows and oceanic flows occurs near to the coastline, limiting the motion of the water. This is the origin of *Kelvin waves*. If one considers the deformation of the free surface of the oceans, one can see that gravity acts as a restoring force, giving rise to a ‘gravity wave’ [7]. When influenced by the Earth’s rotation, these waves are called ‘modified waves’ [8].

Let us consider the following geometry: a South–North current, with a coast on its right (East). The coast is assumed to be a vertical wall, the water height being denoted  $h_0 + h(x, y, t)$ . The Coriolis force usually deflects a South–North current towards the East, i.e. towards the coast. Hence, water gathers close to the coast, and gives rise to a West–East horizontal pressure gradient counterbalancing the Coriolis force. The equations describing the gravity waves are the linearized Euler and continuity equations [7]:

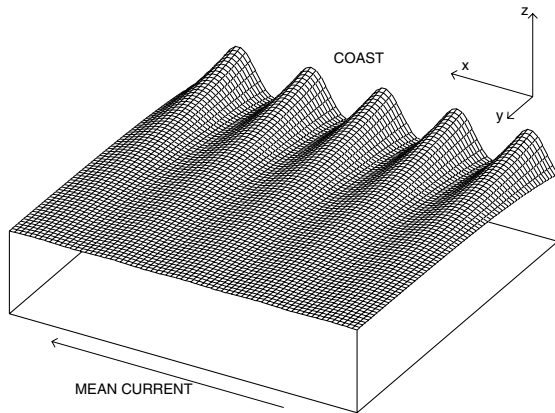
$$\frac{\partial u}{\partial t} = -g \frac{\partial h}{\partial x} + f v \quad \frac{\partial v}{\partial t} = -g \frac{\partial h}{\partial y} - f u \quad \frac{\partial h}{\partial t} = -h_0 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right). \quad (5.3)$$

Taking  $(Oy)$  perpendicular to the coast, and considering a solution describing the above situation, i.e.  $v = 0$ ,  $u = \xi(y) \exp i(\omega t - kx)$ ,  $h = \eta(y) \exp i(\omega t - kx)$ , one obtains

$$u = u_0 \exp \left( -\frac{fy}{\sqrt{gh_0}} \right) \exp i(\omega t - kx) \quad h = u \sqrt{\frac{h_0}{g}}. \quad (5.4)$$

The dispersion relation is given by  $\omega^2 = gh_0 k^2$ , as for usual gravity waves. The characteristic length  $L \equiv \sqrt{gh_0}/f$  is the *Rossby radius of deformation*. At a middle latitude  $\lambda \sim 45^\circ$ , one finds  $L \sim 2200$  km for  $h_0 \sim 5$  km, while for a shallow sea, i.e.  $h_0 \sim 100$  m, one has  $L \sim 300$  km. The surface shape generated by the Kelvin waves is plotted in figure 4. One can notice that the surface undulation is trapped in the vicinity of the coast, and its spatial extent in the direction of the ocean is typically of order  $L$ .

The Kelvin waves are in fact easily observed, since the currents generated by tides are influenced by the Coriolis force and give rise to them. As a consequence, the coast is always to the right of the flow direction (in the Northern Hemisphere). On the scale of the oceanic basin, mean movements are in this case an anticlockwise rotation around a point called the *amphidromic point*. This geometry is found in many places over the globe, the rotation being clockwise in the Southern Hemisphere [8].



**Figure 4.** Surface shape generated by a Kelvin wave.

## 6. Conclusion

Coriolis force effects become important as soon as the spatial extent of the flow is important ( $Ro \propto 1/L$ ). This is the reason why the Earth's rotation has a considerable influence on the dynamics of the atmosphere and the oceans. We have presented in this paper several simple examples of geophysical fluid dynamics. We hope that it will help mechanics teachers to illustrate inertial effects with simple but physically relevant examples.

## Acknowledgment

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